PAPER CODE NO.
PHYS393

UNIVERSITY OF
LIVERPOOL

JANUARY EXAMINATIONS 2008

Bachelor of Science: Year 3
Master of Physics: Year 3
Master of Physics: Year 4

STATISTICAL AND LOW TEMPERATURE PHYSICS

TIME ALLOWED: Three hours

INSTRUCTIONS TO CANDIDATES

Answer all questions.

Question 1 carries 50% of the total marks.

Questions 2 and 3 each carry 25% of the total marks.

The marks allotted to each part of a question are indicated in square brackets.

In the event of a student answering both parts of an either/or question and not clearly crossing out one answer, only the answer to part (a) of the question will be marked.

1.

(a) A system of $N = 4$ distinguishable particles occupies energy states $0, \varepsilon, 2\varepsilon, 3\varepsilon, 4\varepsilon, 5\varepsilon\ldots$ The total energy of the system is $U = 5\varepsilon$.

(i) Write out the 6 possible distributions. **[2]**

(ii) Evaluate the number of microstates for each distribution. **[2]**

(iii) Evaluate the mean populations of the states. **[2]**

A similar system of particles ($N=4$, $U=5\varepsilon$) occupies the same set of energy states but in this case the particles are indistinguishable. Multiple occupation of an energy state is allowed.

(iv) Evaluate the mean population of the states. **[2]**

(b) N atoms bound into a solid system at temperature T can each exist in states of energy $\varepsilon/2$ and $3\varepsilon/2$.

(i) Write an expression for the Partition function of the atoms. **[2]**

(ii) Using the bridge relation

$$U = NkT^2 . \frac{\partial}{\partial T}(\ln Z)$$

or otherwise, show that the internal energy U can be written as

$$U = \frac{N\varepsilon}{2} + \frac{N\varepsilon \exp(-\varepsilon/kT)}{[1 + \exp(-\varepsilon/kT)]} .$$ **[2]**

(iii) Derive limiting values of U as $T \to 0$ and as $T \to \infty$. **[2]**

(iv) Sketch a graph of U versus T. **[2]**

(v) Without further differentiation sketch a graph of C_V versus T. **[1]**

1.

(c) The Maxwell – Boltzmann distribution of speeds of molecules n(v) in a box containing N molecules of mass m at a temperature T can be written

$$n(v) = C.\ v^2\ .\ \exp(-mv^2/2kT)$$

where C is a constant.

(i) Draw the distribution n(v) versus v and mark the most probable speed v_p. [2]

(ii) Derive an expression for v_p. [2]

(iii) Derive an expression for the mean square speed v^2_m. [2]

(iv) Evaluate the mean energy of a monatomic molecule at T = 300K. [1]

(v) Make a reasoned estimate of the mean energy of a diatomic molecule at T = 300K. [2]

The following integral relations may be used.

$$I_n = \int_0^\infty x^n \exp(-bx^2)dx \qquad I_n = ((n-1)/2b)I_{n-2.} \qquad I_1 = 1/2b. \qquad I_0 = ½(\pi/b)^{1/2}$$

(d) A system of particles occupies a set of quantised energy states. The system is at temperature T.

The probability of occupation of a state of energy ε is f(ε).

(i) Write an expression for f(ε) if the particles are fermions. [1]

(ii) Write an expression for f(ε) if the particles are bosons. [1]

(iii) What is the cause of the difference between these cases? [2]

Draw a set of energy states and indicate the populations of these states in the condition of low temperature (T→ 0K).

(iv) for fermions. [2]

(v) for bosons. [2]

UNIVERSITY OF LIVERPOOL

1.

(e) (i) Draw a P – T phase diagram for ^{4}He in the temperature range $0 < T < 5K$. **[2]**

(ii) Label the phases in the diagram. **[2]**

(iii) Describe the experimental results for the measurement of viscosity of ^{4}He in this temperature range. **[2]**

(iv) Explain how these results can be understood. **[2]**

(f) What is meant by a superconductor? **[2]**

Explain how the microscopic BCS theory explains this behaviour. The explanation should include the concepts of critical temperature, Bose condensation and Cooper pairs. **[2,2,2]**

2. Answer **either** 2(a) **or** 2(b)

(a) A cube of side L contains N free conduction electrons.

(i) Write conditions for quantised states of the electrons in terms of their wavevector components k_x, k_y,k_z. **[2]**

(ii) Show the representation of these quantised states in k_x, k_y, k_z space. **[3]**

(iii) Show that the number of states with wavevectors in the range k to k + dk is given by

$$g(k)dk = 2.V.\frac{4\pi k^2\ dk}{(2\pi)^3}$$ **[3]**

where the volume $V = L^3$ and the spin degeneracy of the electrons is included.

(iv) Write the relation between the wavevector k and the energy ε. **[1]**

(v) Show that the number of states with energy between ε and ε +dε is

$$g(\varepsilon)d\varepsilon = 2.V.\frac{(2m/\hbar^2)^{3/2}}{(2\pi)^2}.\varepsilon^{1/2}d\varepsilon.$$ **[3]**

(vi) Show that at T = 0K the Fermi energy μ(0) is given by

$$\mu(0) = \hbar^2.\frac{(3\pi^2 N/V)^{2/3}}{2m}.$$ **[4]**

(vii) Evaluate μ(0) for copper which has fcc structure and a lattice constant $a = 3.61 \times 10^{-10}$m assuming that each copper atom provides one electron to the conduction band. **[3]**

2.

(a)

(viii) The energy U of N conduction electrons at temperature T can be written

$$U(T) = \frac{3N\mu(0)}{5} + \frac{\pi^2.(kT)^2.3N}{12\mu(0)}$$

where in this equation k is the Boltzmann constant.
Evaluate the electronic heat capacity C_V(electrons) of a molar quantity of copper at 4.2K. **[3]**

(ix) The molar lattice heat capacity C_V(lattice) can be written as

$$C_V(\text{lattice}) = 234N_A k(T/\theta_D)^3$$

where N_A is the Avogadro number, k is the Boltzmann constant and θ_D the Debye temperature. For copper $\theta_D = 343$K.

Evaluate the total heat capacity at 4.2K. **[3]**

2.

(b) The density of states for quantised electromagnetic waves in a cavity of volume V can be written in terms of the wavevector k as

$$g(k)dk = 2.\frac{V.4\pi k^2 dk}{(2\pi)^3}$$

(i) Write an expression relating frequency ν and wavevector k for the radiation. **[2]**

(ii) Show that the density of states can be written in terms of the frequency ν as

$$g(\nu)d\nu = \frac{8\pi V \nu^2 d\nu}{c^3}$$

[3]

where c is the speed of light.

(iii) The energy contained in the frequency interval $\nu \rightarrow \nu + d\nu$ is given by

$$\varepsilon(\nu)d\nu = \frac{8\pi V \nu^2 d\nu . h\nu.}{c^3.[\exp(h\nu/kT) - 1]}$$

Explain the terms hν and $1/[\exp(h\nu/kT) - 1]$. **[1], [2]**

(iv) Deduce the limits of $\varepsilon(\nu)$ as $\nu \rightarrow 0$ and as $\nu \rightarrow \infty$. **[2], [2]**

(v) Sketch graphs of $\varepsilon(\nu)$ versus ν for temperatures T_1 and $2T_1$. **[1], [1]**

(vi) Show that the frequency ν_m corresponding to maximum intensity is given by the equation

$$\exp(x) = \frac{3}{(3-x)}$$ **[4]**

where $x = (h\nu_m/kT) = 2.82$.

2.

(b)

vii) Evaluate ν_m for temperatures $T_1 = 6000K$ and $T_2 = 2.9K$. Identify the part of the electromagnetic spectrum in which ν_m lies for each temperature. **[2],[2]**

(viii) Discuss any cosmological significance of the 2.9K distribution. **[3]**

3. Answer **either** 3(a) **or** 3(b).

(a) The element niobium (Nb) is a superconductor with a critical temperature of $T_c = 9.5K$ and a resistivity at 300K of $\rho(300) = 1.3 \times 10^{-7}\ \Omega m$. The element copper (Cu) is not a superconductor and has a resistivity at 300K of $\rho(300) = 1.3 \times 10^{-8}\ \Omega m$.

(i) Sketch graphs of resistivity ρ versus temperature T for Nb and for Cu over the temperature range $0 \rightarrow 300K$. Indicate the nature of the current carriers on the graphs. **[2], [2]**

(ii) Using the ideas of the microscopic theory of superconductivity, explain why Nb has a higher resistivity than Cu at 300K. **[3]**

(iii) Magnetic fields B greater than a critical value B_c destroy superconductivity. Explain how this can be understood. **[2]**

(iv) The variation of B_c with temperature T is given by

$$B_c(T) = B_c(0)\ [1 - (T/T_c)^2]$$

where $B_c(0)$ is the critical field at 0K.

Sketch a graph of B versus T for a superconductor and mark the superconducting and normal regions. **[2]**

(v) For Nb the critical field $B_c(0) = 0.198T$.
Evaluate the critical field at 4.2K. **[2]**
Evaluate the critical temperature in a field of 0.10T. **[2]**

(vi) Describe the Meissner – Oschenfeld effect. **[3]**

(vii) Sketch graphs of (-M) versus applied field H for Type I and for Type II superconductors. **[2], [2]**

(viii) Explain the physical state of the Type II superconductor in each part of the (-M) versus H graph. **[3]**

3.

(b) (i) Write a brief account of superfluid He^3. [5]

(ii) Identify the techniques for cooling (a) to 77K, (b) to 4.2K, (c) to 2K, (d) to $5x10^{-3}$K, (e) to 5×10^{-6}K. [5]

(iii) Describe Pomeranchuk cooling. [5]

(iv) Describe the Dilution refrigerator. [5]

(v) In an adiabatic demagnetization process a sample in contact with a heat bath at $2x10^{-3}$K is magnetized with a field of 12T. The sample is then thermally isolated and demagnetized leaving only a residual field of 6×10^{-3}T. Suggest a suitable sample material and evaluate the final temperature. Why might the final equilibrium temperature be slightly higher than your evaluated temperature? [5]

CONSTANTS

Speed of light in vacuum	c	=	$3.00 \times 10^{8}\ \mathrm{ms^{-1}}$
Permeability of vacuum	μ_0	=	$4\pi \times 10^{-7}\ \mathrm{Hm^{-1}}$
Permittivity of vacuum	ε_0	=	$8.85 \times 10^{-12}\ \mathrm{Fm^{-1}}$
Elementary charge	e	=	$1.60 \times 10^{-19}\ \mathrm{C}$
Planck constant	h	=	$6.63 \times 10^{-34}\ \mathrm{Js}$
Avogadro constant	N_A	=	$6.02 \times 10^{23}\ \mathrm{mol^{-1}}$
Boltzmann constant	k	=	$1.38 \times 10^{-23}\ \mathrm{JK^{-1}}$
Gas constant	R	=	$8.31\ \mathrm{JK^{-1}mol^{-1}}$
Unified atomic mass constant	m_u	=	$1.66 \times 10^{-27}\ \mathrm{kg}$
		=	$931\ \mathrm{MeVc^{-2}}$
Electron mass	m_e	=	$9.11 \times 10^{-31}\ \mathrm{kg}$
Proton mass	m_p	=	$1.67 \times 10^{-27}\ \mathrm{kg}$
Gravitational constant	G	=	$6.67 \times 10^{-11}\ \mathrm{Nm^2kg^{-2}}$
Acceleration due to gravity	g	=	$9.8\ \mathrm{ms^{-2}}$
Bohr magneton	μ_B	=	$9.27 \times 10^{-24}\ \mathrm{JT^{-1}}$